Design of Experiments with Two-Level and Four-Level Factors

by
Bruce Ankenman
Department of Industrial Engineering and Management Sciences
Northwestern University
Evanston, IL 60208

March 29, 1999

ABSTRACT

Practitioners, familiar with the design of two-level fractional factorials, are often frustrated when confronted with factors that have more than two levels. This article provides a guide for the inclusion of four-level factors into standard two-level factorial designs. Tables are presented to allow for the design of experiments with two-level and four-level factors using the same types of experimental design criteria commonly used for designing two-level fractional factorials. The concepts of resolution, aberration, and foldover are explained in the context of experiments with two-level and four-level factors. Several new minimum aberration designs that are not in the literature are listed in the tables and new follow-up designs for the recommended experiments are also provided.

Introduction

Many experimental design textbooks and software packages emphasize the use of factorial and fractional factorial designs where all factors in the experiment have two levels, often called 2^{k-p} designs, where *k* is the number of factors, *p* is the degree of fractionation, and 2^{k-p} is the number of runs. There are many good reasons for using only two levels per factor, including: reducing the size of the experiment, allowing for sequential experimentation, taking advantage of the relatively simple confounding properties of two-level fractional factorials, and allowing for simple graphical analysis of main effects and interactions (see Box, Hunter, and Hunter, 1978).

Bisgaard (1997a) discusses the advantages of two-level fractional factorials for technological experiments where all the factors are quantitative, meaning that the levels are measured or metered amounts that are usually continuous, but at least ranked. It is true that technological experiments often have only quantitative factors, however, it is not uncommon for technological experiments to also include factors that are qualitative in nature such as raw material type, supplier name, or die configuration. There are often more than two levels of such factors and since the levels cannot be ordered in any meaningful way, leaving out levels of such factors provides no information about the response's behavior at the omitted levels. In order to include multi-level qualitative factors, but maintain some of the useful characteristics of two-level fractional factorial experiments, a common practice is to use a simple coding scheme, like the one in Table 1, to convert two columns, A and B, of a 2^{k-p} design into a single column, X, for a four-level factor. As the table shows, the coding scheme calls for level one of the four-level factor whenever both A and B are at the low level. The second level of the four-level factor is used whenever A is at the high level and B is at the low level and similarly for the other two levels. Many authors have described this procedure including Taguchi (1987, p.205) and Montgomery (1997, p.464).

Two-Level Factors			Four-Level Factor
A	B		X
-1	-1	→	1
1	-1	→	2
-1	1	→	3
1	1	→	4

Table 1. A Coding Scheme for Converting 2 Columns, A and B, from a Two-Level Fractional Factorial into a Single Column, X, for a Four-Level Factor.

The purpose of this article is to guide experimenters in the design of experiments with two-level and four-level factors. If in general there are *m* four-level factors and *n* two-level factors in an experiment, the experiment can be called a 4^m2^{n-p} design, where *p* is again the degree of fractionation and 4^m2^{n-p} is the number of runs. In the following discussions, some knowledge of the design of two-level fractional factorial experiments will be assumed. For more information on the design techniques for 2^{k-p} designs see Box, Hunter, and Hunter (1978, Ch. 12) or Montgomery (1997, Ch. 9).

In the next section, an example from the chemical additive industry is used to demonstrate the experimental design problems encountered in experiments with two-level and four-level factors and to review the design criteria that are usually considered in designing two-level fractional factorial experiments. The third and fourth sections extend these design criteria to the case of experiments with two-level and four-level factors. The fifth and sixth sections introduce a set of tables that have been developed through computer search to provide the best 4^m2^{n-p} designs according to the criteria for specified values of *m* and *n*. Tables are provided for experiments with 8, 16, and 32 runs. Although Wu and Zhang (1993) have previously listed some of these designs, the tables also provide follow-up designs that have not appeared in the literature. The use of the tables for finding an initial and a follow-up design is then demonstrated by redesigning the chemical additive experiment.

An Example

The following example comes from the chemical additive industry. The purpose of the experiment is to improve the performance of an automatic transmission by formulating an additive package that is blended into a base oil to create transmission fluid. The fluid is then tested in an automatic transmission that uses a friction material as an integral part of the transfer of torque from the engine to the drive wheels. Among the many ingredients of the chemical additive package are detergents, dispersants and friction modifiers. The factors of the experiment are given in Table 2 and it is clear that there are two four-level factors and three two-level factors.

Factor	Levels
Friction Material	Type 1,2,3, and 4
Base Oil	Type 1,2,3, and 4
Dispersant	Low and High Amount
Detergent	Low and High Amount
Friction Modifier	Type 1 and 2

Table 2. The Factors and Levels for a Transmission Improvement Experiment.

The design strategy that is used for this experiment is to start by designing a reasonably sized (16-run) 2^{k-p} design with good characteristics. This design, called the base design, is then modified to accommodate the four-level factors by using the coding scheme in Table 1 twice. First, the coding is used to convert two columns of the design into a single column for Friction Material and then it is applied again to convert two different columns into a column for Base Oil. The resulting design is a $4^2 2^{3-3}$ design.

Run	A	B	C	D	E	F	G
1	-1	-1	-1	-1	-1	-1	-1
2	1	-1	-1	-1	1	-1	1
3	-1	1	-1	-1	1	1	-1
4	1	1	-1	-1	-1	1	1
5	-1	-1	1	-1	1	1	1
6	1	-1	1	-1	-1	1	-1
7	-1	1	1	-1	-1	-1	1
8	1	1	1	-1	1	-1	-1
9	-1	-1	-1	1	-1	1	1
10	1	-1	-1	1	1	1	-1
11	-1	1	-1	1	1	-1	1
12	1	1	-1	1	-1	-1	-1
13	-1	-1	1	1	1	-1	-1
14	1	-1	1	1	-1	-1	1
15	-1	1	1	1	-1	1	-1
16	1	1	1	1	1	1	1

Table 3. An Experimental Design for a 2^{7-3} design, where E=ABC, F=BCD, and G=ACD.

Since each four-level factor will require two columns and each two-level factor will require one column, the base design must have a total of seven columns. Note that, in general, the base design for a $4^m 2^{n-p}$ design will be a 2^{k-p} design where $k=2m+n$. For this case, the standard design generators (see Box, Hunter, and Hunter, 1978, p.410) for a 16-

run, two-level fractional factorial design with seven columns (a 2^{7-3} design) are: E=ABC, F=BCD, and G=ACD. For this design, which is shown in Table 3, the *defining relation* is I=ABCE=BCDF=ACDG=ADEF=BDEG=ABFG=CEFG. The defining relation lists all of the factor interactions that cannot be estimated by the design because they are held constant. Each of the groups of letters in the defining relation is called a *word* and it designates a set of columns that when multiplied together form the identity column, I, which is a column of all ones. The words in the defining relation determine the severity of the confounding in the design.

The most common criterion for choosing a particular set of design generators for a 2^{k-p} design is called *resolution*. The resolution of a design is usually designated by a Roman numeral and is easily found by counting the number of letters in the shortest word in the defining relation. For the design in Table 3, all the words in the defining relation are of length four so the resolution of this design is IV. Resolution seeks to reduce the confounding between lower order effects. A resolution IV design is guaranteed to have main effects confounded only with interactions of order three or higher. A resolution III design would only guarantee that main effects are not confounded with each other, but would have some main effects confounded with two-factor interactions. Thus, for a given number of runs, resolution is usually maximized.

Aberration, introduced by Fries and Hunter (1980), is a concept that is similar to, but more discriminating than resolution. For example, aberration recognizes that a resolution III design with only one three-letter word in its defining relation is preferable to a design with the same number of runs that has two three-letter words in its defining relation. For comparing two designs, the best way to apply the concept of aberration is to write a vector, called the word length pattern (WLP), that records the frequency of the word lengths in each defining relation. The first element in the vector records the number of single letter words (excluding I); the second element records the number of two-letter words, and so forth. Since designs of resolution II or lower are usually not considered, some authors omit the first two places of the WLP. In this article, the first two elements will be retained as placeholders so that the *i*th element of the WLP refers to the number of *i*-letter words. Suppose, for example, that the defining relation for a 16-run, two-level fractional factorial with six factors is I=BCE=BCDF=DEF. Since this relation has two three-letter words and one four-letter word, it has a WLP of (0,0,2,1,0,0). To compare this design to a second design with defining relation I=ABE=CDF=ABCDEF and subsequent WLP of (0,0,2,0,0,1), one would note that from left to right the first location where the vectors differ is in the fourth place. Since the second design has a lower number at this location, it has lower aberration than the first design. For a given design size and number of factors, a design that has the lowest aberration is called a minimum aberration design. A minimum aberration design always has maximum resolution, but is usually not unique.

Returning now to the design of the transmission fluid experiment, the design in Table 3 that has been selected as the base design has resolution IV, has a WLP of (0,0,0,7,0,0), and is a minimum aberration design for 2^{7-3} designs. The next step is to modify the base design by choosing two pairs of columns to be converted into four-level columns for Friction Material and Base Oil. Without any criteria to follow, a practitioner might choose A and B to convert to the factor, X, for Friction Material and C and D to convert to factor Y for Base Oil. This is done in Table 4.

Many questions can now be asked about the design in Table 4. What is its resolution? Does it have minimum aberration? These questions will be addressed in the next section along with another important question: If this experiment is run and there is excessive confounding among significant effects, what follow-up designs can be run to reduce the confounding of the experiment?

Run	Fr. Mat. X	Base Oil Y	Disp. E	Det. F	Fr. Mod. G
1	1	1	-1	-1	-1
2	2	1	1	-1	1
3	3	1	1	1	-1
4	4	1	-1	1	1
5	1	2	1	1	1
6	2	2	-1	1	-1
7	3	2	-1	-1	1
8	4	2	1	-1	-1
9	1	3	-1	1	1
10	2	3	1	1	-1
11	3	3	1	-1	1
12	4	3	-1	-1	-1
13	1	4	1	-1	-1
14	2	4	-1	-1	1
15	3	4	-1	1	-1
16	4	4	1	1	1

Table 4. An Experimental Design for Two Four-Level Factors and Three Two-Level Factors.

Resolution and Aberration when using Two-Level and Four-Level Factors

In order to understand resolution and aberration for experiments with two-level and four-level factors, it must be noted that for any factor that appears in an experiment at *m* levels, there are *m-1* degrees of freedom associated with the main effect of that factor. Thus, there is one degree of freedom associated with the main effect of each of the two- level factors, but there are three degrees of freedom associated with the main effect of each of the four-level factors. Below each two-level factor label (E, F, and G) in Table 4, there is a column that shows the contrast that is associated with the degree of freedom for the main effect of that factor. However, as Addelman (1972), Bisgaard (1994), Bisgaard (1997b) note, the three contrasts associated with the main effect of each four-level factor are the two columns from the base design that were used in the coding of the factor and the column associated with the interaction between these two columns. The three contrasts associated with factor X, Friction Material, in the design of Table 4 are then columns A and B in the base design **and** AB, the column formed by multiplying column A and B together. Since these are now the main effect contrasts of X, they will be renamed as follows: $A=X_1$, $B=X_2$, and $AB=X_3$. Similarly $C=Y_1$, $D=Y_2$, and $CD=Y_3$ are the contrasts associated with the main effects of factor Y, Base Oil.

Recall that the defining relation for the base design in Table 3 was I=ABCE=BCDF=ACDG=ADEF=BDEG=ABFG=CEFG. When the design is modified to include the four-level factors, the contrasts AB and CD are now no longer two-factor interactions but are main effects of the four-level factors. The relationships $AB=X_3$ and $C=Y_1$ above can be used to modify the first word in the defining relation ABCE to X_3Y_1E to account for the four-level factors. By modifying each word in the same way, the defining relation for the design in Table 4 can be shown to be $I=X_3Y_1E=X_2Y_3F=X_1Y_3G=X_1Y_2EF=X_2Y_2EG=X_3FG=Y_1EFG$. Notice that the WLP of this design is now (0,0,4,3,0). This is a resolution III design since some two-factor interactions are confounded with some main effects. By making the modifications to the defining relation, the simple method of determining resolution can be used since the length of the smallest word in the defining relation is now equal to the resolution of III.

The question of whether or not this is a minimum aberration design can be answered by comparing the WLP of this design with all other possibilities for the generators: E=?, F=?, and G=?. When this is done, it is found that the minimum aberration design for a 16-run design with two four-level factors and three two-level factors also has a WLP of (0,0,4,3,0). Thus, the design in Table 4 is a minimum aberration $4^2 2^{3-3}$ design. Wu and Zhang (1993) already have produced a list of many minimum aberration designs for experiments with two-level and four-level factors. They use a slightly more complicated definition of minimum aberration that gives higher priority to interactions between two-level factors and lower priority to interactions that involve four-level factors. Except when the four-level factor is quantitative or in cases where there is some prior knowledge of effects, I see no compelling reason to assign a hierarchy among the effects of the four-level factor. I thus prefer the definition as explained above which uses the frequencies of the word lengths regardless of the origin of the word.

Since the design in Table 4 is a minimum aberration design, the only remaining question is whether there is a good follow-up for this design. The next section describes how follow-up designs in two-level fractional factorial experiments are sometimes used to reduce confounding and thus provide clearer information about the relationships between the factors and the response. Follow-up designs for experiments with two-level and four-level factors will then be addressed.

Follow-up Designs for Reducing Confounding in Factorial Experiments

When dealing with two-level fractional factorials, there is a theorem provided in most experimental design textbooks stating that for any resolution III design, there is a follow-up design with the same number of runs that, when combined with the original design, will produce a design of resolution IV or higher. A common follow-up design, which always possesses this property, is called a *complete foldover* design because it takes the levels of all of the factors in the original design and switches them.

When a design is folded over on only some of the factors, called *folding factors*, only the signs of the folding factors are switched in the follow-up design and the other factors retain the same signs that they had in the initial design. This is often done to isolate certain interaction effects. There is a general rule that can be applied to determine the defining relation of any two-level fractional factorial design that has been created by combining an initial 2^{k-p} design with a foldover on any number of folding factors. The rule states that the defining relation of the combined design is the same as the defining relation of the initial design except that all words that contain an odd number of the folding factors are eliminated. For example, suppose a 2^{6-2} design has the defining relation I=ABCE=CDF=ABDEF. If the design is completely folded over, the combined design will have defining relation I=ABCE since the other two words (CDF and ABDEF) contain an odd number of folding factors. If the design were to be folded over on only factors E and F, then the defining relation for this combined design would be I=ABDEF, which would have higher resolution (resolution V) than the combined design from the complete foldover (resolution IV). For more information on foldover designs for two-level fractional factorials see Montgomery (1997) and Montgomery and Runger (1996).

When four-level factors are included in a design, the concept of foldover must be slightly modified because there is no longer a clear opposite for each level of the four-level factors. For example, suppose that level 1 of a four-level factor is used in the first run of an initial design. What level of the four-level factor should be used in the first run of a follow-up design that is using the four-level factor as a folding factor? The most obvious extension of the foldover concept is to think about folding over columns of the base design and not think about folding over factors in the actual design. However, if columns A, B, and AB are associated with a four-level factor in an initial design, they must also be associated with that factor in combined design. It is then impossible to fold on all three of the columns in the base design and retain the relationship A*B=AB, so only two of the three columns can be used when folding. For simplicity, it is easier to think about folding over on either one or both of the coding columns, A and B, and always allowing the third column AB to be determined by the other two (as it must be). The results of this simplification is that when both A and B are folded, AB will have the same signs in the follow-up design as it had in the initial design. When either only A or only B is folded over, the column AB will change signs in the follow-up.

As an example, observe the initial design in Table 5 that shows an experiment with one four-level factor, X, and three two-level factors, C, D, and E, where the generators for the base design are D=ABC and E=AC. The table also shows the columns, A and B, from the base design that were used in the coding. One follow-up design can be defined by folding over all two-level factors and both coding columns from the base design and then applying the coding scheme to columns A and B to produce X. This design is shown in Table 5.

The defining relation for the base design is I=ABCD=ACE=BDE, which makes the defining relation for the initial design, $I=X_3CD=X_1CE=X_2DE$, clearly a resolution III design. When the base design is folded over all columns, the defining relation of the combined base design becomes I=ABCD, and thus is a resolution IV design. However, by using the relationship X_3=AB, the defining relation of the combined design with the four-level factor, X, and the two-level factors C, D, and E can be determined to be I= X_3CD, which has resolution III, the same resolution as the initial design. In fact, there is no set of folding columns for the initial design in Table 5 that will produce a resolution IV combined design. This may be a surprising result since for two-level factorials, the theorem guarantees that any resolution III design can be folded over to produce a resolution IV design. There are, however, some experimental designs with two-level and four-level factors that can be folded over on certain factors to increase their resolution. In the next section, tables are introduced that show 4^m2^{n-p} designs of resolution III that can be followed-up to provide combined designs of resolution IV or higher.

	Coding columns from the base design		An experiment with one four-level factor and three two-level factors				
Run	A	B	X	C	D	E	
1	-1	-1	1	-1	-1	1	Initial Design
2	1	-1	2	-1	1	-1	
3	-1	1	3	-1	1	1	
4	1	1	4	-1	-1	-1	
5	-1	-1	1	1	1	-1	
6	1	-1	2	1	-1	1	
7	-1	1	3	1	-1	-1	
8	1	1	4	1	1	1	
9	1	1	4	1	1	-1	Follow-up Design Folded on A & B
10	-1	1	3	1	-1	1	
11	1	-1	2	1	-1	-1	
12	-1	-1	1	1	1	1	
13	1	1	4	-1	-1	1	
14	-1	1	3	-1	1	-1	
15	1	-1	2	-1	1	1	
16	-1	-1	1	-1	-1	-1	

Table 5. A Foldover Design for One Four-Level Factor and Three Two-Level Factors.

Tables for Designing Experiments with Two-Level and Four-Level Factors

The tables in the Appendix give recommended experimental designs and follow-up designs for 8-run, 16-run, and 32-run experiments with as many as three four-level factors and as many as fourteen two-level factors. In the tables, each column provides designs for a different number (*m*) of four-level factors and each row provides designs for a different number (*n*) of two-level factors. The design generators for creating the base design are given in the top left-hand corner of each cell. Columns A and B from the base design are always used for creating the first four-level column, columns C and D are always used for creating the second four-level column, and columns E and F are always used for creating the third four-level column. The contrasts from the base design that are associated with the main effects of the four-level factors are provided in the column heading. Note that in some cases there are generators for the coding columns E and F also provided in the column heading. In the lower right hand corner of each cell are the instructions for folding over the base design to produce a combined design of resolution IV or higher whenever it is possible.

The computer search for designs used a *Mathematica* program to calculate the WLP of the recommended set of generators and compare it with all other possible sets of generators. A similar program was used to provide the WLP for every possible foldover design to a given design or set of designs. Often the search for a single design would run for several days on a Pentium 133MHz machine indicating that the number of potential designs is often extremely large.

The primary criterion for choosing the generators for the recommended base design is minimum aberration of the initial 4^m2^{n-p} design. If the minimum aberration design for a given *m* and *n* is less than resolution III, then that design is designated as "Not Available." In a few cases where the minimum aberration design cannot be folded over to produce a design with resolution IV, a design with slightly higher aberration that can be folded over to a resolution IV design is recommended. There are also instances of 32-run experiments where the minimum aberration design is not known because the computer search was too large. In these cases, a resolution III design that can be folded over to produce a resolution IV or higher design is provided as the initial design. When the initial 4^m2^{n-p} design is a minimum aberration design, an asterisk is placed on the final generator for the recommended base design.

In many cases, a minimum aberration design can be folded over to produce a combined design that is also a minimum aberration design. These minimum aberration combined designs are designated by the asterisk next to the "Fold on . . ." directions in the lower right hand corner of the cell. When the combined design is not minimum aberration, but is the best design that can be achieved by folding over any minimum aberration design of that size, the combined design is shown with two asterisks. If there is no way to produce a resolution IV design by folding over a resolution III design, the table provides the minimum aberration design and then shows that a resolution IV design is either "Not Available" at all or "Not Available by Foldover" of any resolution III design.

Since most minimum aberration designs are not unique and often folding on one set of factors is equivalent to folding on a different set of factors, there are many ways to achieve designs that are equivalent to the designs provided in these tables.

Applying the Tables to the Transmission Example

For the transmission example, the design in Table 4 is indeed a minimum aberration design, however, there is no combination of folding factors that will yield a combined design with resolution IV or higher. On Table A2 for 16-run designs, the design generators (E=AD, F=BC, and G=ABCD) are provided for a base design with defining relation, I=ADE=BCF=ABCDG=ABCDEF=BCEG=ADFG=EFG (see Table 6). Using $A=X_1$, $B=X_2$, $AB=X_3$, $C=Y_1$, $D=Y_2$, and $CD=Y_3$, the defining relation is modified for the four-level factors to become $I=X_1Y_2E=X_2Y_1F=X_3Y_3G=X_3Y_3EF=X_2Y_1EG=X_1Y_2FG=EFG$. This design has a WLP of (0,0,4,3,0) which, like the design in Table 4, is a minimum aberration design. When this design is folded over on factors A and F as directed in the

Table A2, the combined base design has defining relation I=ABCDEF=BCEG=ADFG (see Table 6). The defining relation becomes I= $X_3Y_3EF=X_2Y_1EG=X_1Y_2FG$ after modification for the four-level factors. The WLP pattern for this design is then (0,0,0,3,0). This combined design is not only a resolution IV design, but is also the minimum aberration design for a 32-run experiment with two four-level factors and three two-level factors.

Conclusion

The ideas of resolution, aberration, and foldover, common in the design of two-level fractional factorial experiments, have been extended and explained in the context of experiments with two-level and four-level factors. Tables have been provided to allow practitioners to quickly design experiments with two-level and four-level factors. The tables usually recommend minimum aberration designs of resolution III or higher that can be folded over on certain factors to achieve a combined design of resolution IV or higher. If the minimum aberration design cannot be folded over to achieve resolution IV, a resolution III design which can be folded over to achieve resolution IV is recommended when available. These tables then allow the practitioner to produce experimental designs for experiments with two-level and four-level factors that have confounding and foldover properties similar to the standard two-level fractional factorial designs that are so widely known and used.

Acknowledgement

This research was sponsored in part by the Morris E. Fine Junior Professorship in Materials and Manufacturing. The author would like to thank Elizabeth Schiferl and Daniel Meyer for helpful discussions leading to this work. I also appreciate the efforts of the referees and their helpful comments.

	Coding columns from the base design				An experimental design for the transmission example					
Run	A	B	C	D	Fr. Mat. X	Base Oil Y	Disp. E	Det. F	Fr. Mod. G	
1	-1	-1	-1	-1	1	1	1	1	1	Initial Design
2	1	-1	-1	-1	2	1	-1	1	-1	
3	-1	1	-1	-1	3	1	1	-1	-1	
4	1	1	-1	-1	4	1	-1	-1	1	
5	-1	-1	1	-1	1	2	1	-1	-1	
6	1	-1	1	-1	2	2	-1	-1	1	
7	-1	1	1	-1	3	2	1	1	1	
8	1	1	1	-1	4	2	-1	1	-1	
9	-1	-1	-1	1	1	3	-1	1	-1	
10	1	-1	-1	1	2	3	1	1	1	
11	-1	1	-1	1	3	3	-1	-1	1	
12	1	1	-1	1	4	3	1	-1	-1	
13	-1	-1	1	1	1	4	-1	-1	1	
14	1	-1	1	1	2	4	1	-1	-1	
15	-1	1	1	1	3	4	-1	1	-1	
16	1	1	1	1	4	4	1	1	-1	
17	1	-1	-1	-1	2	1	1	-1	1	Foldover Design on A & F
18	-1	-1	-1	-1	1	1	-1	-1	-1	
19	1	1	-1	-1	4	1	1	1	-1	
20	-1	1	-1	-1	3	1	-1	1	1	
21	1	-1	1	-1	2	2	1	1	-1	
22	-1	-1	1	-1	1	2	-1	1	1	
23	1	1	1	-1	4	2	1	-1	1	
24	-1	1	1	-1	3	2	-1	-1	-1	
25	1	-1	-1	1	2	3	-1	-1	-1	
26	-1	-1	-1	1	1	3	1	-1	1	
27	1	1	-1	1	4	3	-1	1	1	
28	-1	1	-1	1	3	3	1	1	-1	
29	1	-1	1	1	2	4	-1	1	1	
30	-1	-1	1	1	1	4	1	1	-1	
31	1	1	1	1	4	4	-1	-1	-1	
32	-1	1	1	1	3	4	1	-1	-1	

Table 6. An Experimental Design and Follow-up Design for Two Four-Level Factors and Three Two-Level Factors.

References

Addelman, S. (1972). “Recent Developments in the Design of Factorial Experiments”. *Journal of the American Statistical Association* 67, pp. 103-111.

Bisgaard, S. (1997a). “Why Three-Level Designs Are Not So Useful for Technological *Quality Engineering* 9, pp. 545-550.

Bisgaard, S. (1997b). “Accommodating Four-Level factors in Two-Level Factorial *Quality Engineering* 10, pp. 205-206.

Bisgaard, S. (1994). “A Note on the Definition of Resolution for Blocked 2^{k-p} Designs”. *Technometrics* 36, pp. 308-311.

Box, G. E. P.; Hunter, W. G.; and Hunter, J. S. (1978). *Statistics for Experimenters*. John Wiley, New York, NY.

Fries, A.; and Hunter, W. G. (1980). “Minimum Aberration 2^{k-p} Designs”. *Technometrics* 22, pp. 601-608.

Montgomery, D. C. (1997). *Design and Analysis of Experiments*, Fourth Edition. John Wiley, New York, NY.

Montgomery, D. C.; and Runger, G. C. (1996). “Foldovers of 2^{k-p} Resolution IV designs”. *Journal of Quality Technology* 28, pp. 446-450.

Taguchi, G. (1987). *System Of Experimental Design*, Volume 1. Unipub/Kraus International Publications, White Plains, NY.

Wu, C. F. J.; and Zhang, R. (1993). “Minimum aberration designs with two-level and *Biometrika* 80, pp. 203-209.

Key Words: *Minimum Aberration, Foldover, Resolution, Orthogonal Arrays, Fractional Factorial Designs*, *Follow-up Designs*.

Biographical Note: Bruce Ankenman is an Assistant Professor in the Department of Industrial Engineering and Management Sciences at Northwestern University. He is a Member of ASQ.